Calculus
SUMMER REVIEW PACKET

The problems you will find in the following few pages contain concepts in Algebra and Trigonometry that you will encounter in your study of Calculus. The difficulty in understanding Calculus is more theoretical than practical, but a good foundation is essential in attaining correct solutions. The topics chosen for this review are designed to make you more proficient in these areas, so that as they arise in our study of Calculus you will be able to recognize and utilize them in the solving of application problems.

Do not wait until the last week of summer to work on this packet, pace yourself throughout the summer. This packet will count towards your homework grade and you **must** have it **completed** to receive full credit. If you don't understand something, do your best to complete the problem and then make a note in the margin as a reminder to ask the teacher for a better explanation when you are in class.

Bring this packet with you on the first day of class because it will be reviewed the first week of school and all problems will be explained. Then a test will be given after all problems have been reviewed.

Have a safe and enjoyable summer! Your teachers look forward to meeting you in September!

Good luck and success next year. Calculus is Awesome!!!

Enjoy your summer, but keep your mind stimulated.

<u>Binomial Expansion</u>

<u>Pascal's Triangle</u>

Some problems in mathematics involve taking a binomial and raising it to any power. You are most familiar with squaring a binomial ;
ex. $(a + b)^2 = a^2 + 2ab + b^2$. Pascal discovered a clever pattern involving the coefficients of a binomial raised to any power. Observe this **triangle** of numbers :

1	$(a + b)^0$
1 1	$(a + b)^1$
1 2 1	$(a + b)^2$
1 3 3 1	$(a + b)^3$
1 4 6 4 1	$(a + b)^4$
1 5 10 10 5 1	$(a + b)^5$
1 6 15 20 15 6 1	$(a + b)^6$

Thus, $(a + b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$. Also, observe what is happening to the exponents. The variable **a** decreases as the variable **b** increases .
Now suppose your binomial is $(2x + 3y)^3$. In this case , let **$a = 2x$ and $b = 3y$.**
Then, $(2x + 3y)^3 = \mathbf{1}(2x)^3 + \mathbf{3}(2x)^2(3y) + \mathbf{3}(2x)(3y)^2 + \mathbf{1}(3y)^3$.
Now see if you can answer the following questions.

1. Write the coefficients in the next line of Pascal's triangle for $(a + b)^7$.
2. Expand the binomial $(r + s)^4$
3. Expand the binomial $(3x + 4y)^2$
4. Expand the binomial $(a + 2b)^3$, and simplify.
5. Expand the binomial $(a + b)^7$

<u>Domain and Range</u>

When you were first introduced to the definitions of Domain and Range you were told that the domain represents the set of all first coordinates of the ordered pairs for that particular problem , and that the range represents the set of all second coordinates for that problem. On an x-y coordinate plane the domain is the set of x-values and the range is the set of y-values . The variable x is known as the independent variable, while the variable y is called the dependent variable. We call x independent because for many math problems we can arbitrarily choose any real number for x , but the value of y is not *chosen* , it is determined by the value of x. In many math problems the domain is the set of Real numbers, in other words we can choose any real number to represent x. However this is not always the case because for certain problems , choosing a particular value for x will result in either getting an answer for y that is undetermined *or* getting an answer for y that is not a real number. Therefore it is important to understand which values of x you are allowed to use for a particular problem so that the resulting value of y will also produce a real number.

For example, in the equation $y = x^2 + 3x - 4$, you can choose any real number for x and the resulting answer for y will *always* also be a real number. Thus, we can say that the domain is the set of all real numbers . However, determiming the range requires some thought and some work. Even though we know that we will always get a real number answer for y, the range may not include *all* real numbers. If you take a look at the graph of the equation you will find that the curve is a parabola having a vertex of (-1.5,-6.5) . In other words, the graph never goes below the point y= -6.5 . Thus the range for this problem is $y \geq -6.5$.

There are many equations in math where the domain is always all real numbers; every polynomial function has all real #'s as it's domain. However, there are also many functions where the domain is restricted to less than all real's, even if only one number may not be included. Rational functions(fractions) for instance restrict the domain of a function. Consider the equation : $y = \frac{3x+5}{x-4}$. Since we know that division by zero is impossible (it makes no sense) , it follows that $x - 4 \neq 0$,
Thus, $x \neq 4$. Therfore we must eliminate the value of 4 from the domain.
Another common function that restricts the domain is a function involving square roots. Since a negative square root produces an imaginary number, not a real number, we must eliminate any values from the domain that will produce a
negative square root. For example, the function $y = \sqrt{x+5}$ will produce negative answers for any value of $x < -5$. Therefore , the domain for this function is $x \geq -5$.
In the following problems find the domain and/or the range for each equation. A graphing calculator can be used to help you find the range.

6. Find the Domain and Range for $y = 5x + 4$
7. Find the Domain and Range for $y = x^2 + 5x - 6$
8. Find the Domain for $y = \frac{6x-5}{x-5}$
9. Find the Domain and Range for $y = \sqrt{3x+5}$
10. Find the Domain and Range for $y = \sqrt{\frac{x+2}{x-2}}$
11. Find the Domain for $y = \frac{5}{x^2 - 9}$.

Linear Equations

Finding equations of lines using the point-slope method

An important concept in calculus is finding equations of tangent lines to curves. The tangent line is a linear equation that intersects the graph at exactly one point. Usually we write the equation of the line in the form **y = mx + b .** However, if you are are given the slope of the line , **m** , and a point **(x , y)** it is preferable to use the **point-slope formula**,
$\mathbf{y - y_1 = m(x - x_1)}$ and then simplify that equation .
For example; find the equation of the line having slope = -3 and passing through the point (4 , -6) .
Using the pt.-slope formula ,we have $y - (-6) = -3(x - 4)$. Simplifying this and writing it in **y = mx + b form** we get $y = -3x + 6$.
Two other facts to remember : 1) parallel lines have the same slope and 2) perpendicular lines have slopes that are negative reciprocals of each other.
Now , find the equation of each line given the following information .

12. The slope = -2 and the line passes through the point (3, -5) .

13. The slope = $\frac{3}{4}$ and the line passes through the origin.
14. The slope = 0 and the line passes through the point ((6 , -6) .
15. The line passes through the point (3 , -3) and is perpendicular to the line y = x + 4 .
16. The line passes through the point (-1 , -7) and is parallel to the line 2x – 3y = 6 .
17. The line has no slope and passes through the point (6 , 8) .
18. The line has an x-intercept of -2 and y-intercept of 8 .
19. The line is parallel to the x-axis and passes through the point (4, -2) .

New Functions from Old

Just as numbers can be added, subtracted, multiplied, and divided to produce other numbers, **functions** can also be added, subtracted, multiplied, and divided. Furthermore, one can also find functions within functions, known as **composite functions**. Look at these definitions :

1. (f + g)(x) = f(x) + g(x)
2. (f – g)(x) = f(x) - g(x)
3. (fg)(x) = f(x)g(x)
4. (f/g)(x) = f(x) / g(x) , where g(x) ≠ 0

For example,

Let f(x) = $x^2 + 1$ and g(x) = 2x + 4

Then (f + g)(x) = $x^2 + 2x + 5$
(f – g)(x) = $x^2 – 2x – 3$
(fg)(x) = $2x^3 + 4x^2 + 2x + 4$
(f/g)(x) = $\frac{x^2+1}{2x+4}$ where x = -2

The composition of two functions , f and g , is written as f(g(x)) and is defined to consist of all x in the domain of g for which g(x) is in the domain of f.
Consider the two functions f and g from above. The composite f(g(x)) = f(2x+4) which then can be evaluated as $(2x+4)^2 + 1$.
Look at another example :
Let f(x) = 5x + 6 and g(x) = 2 ; then f(g(x)) = f(2) = 5(2) +6 = 16 .

Now try these problems :

Given f(x) = $\sqrt{x}$ and g(x) = sin(x) , find :

20. (f + g)(x)
21. (f – g)(x)
22. (fg)(x)
23. (f/g)(x)
24. f(g(x))

Given f(x) = $x^2 + 4x – 21$ and g(x) = -3 , find :

25. (f + g)(x)

26. (f – g)(x)
27. (fg)(x)
28. (f/g)(x)
29. f(g(x))
30. If g(x) = -4x , find g(f(x))

Trigonometric Identities

In your study of Calculus you will encounter trigonometric as well as logarithmic functions and you will need to use them to solve certain types of application problems. Trig identities can help you simplify some problems thus making it easier to attain solutions. Recall the following identities you learned:

1. $\sin^2\vartheta + \cos^2\vartheta = 1$
2. $1 + \tan^2\vartheta = \sec^2\vartheta$
3. $1 + \cot^2\vartheta = \csc^2\vartheta$
4. $\sin 2\vartheta = 2\sin\vartheta\cos\vartheta$
5. $\cos 2\vartheta = \cos^2\vartheta - \sin^2\vartheta$
6. $\tan 2\vartheta = \dfrac{2\tan\vartheta}{1-\tan^2\vartheta}$

Also, recall the definition for logarithmic functions:
$\log_b x = y$ if and only if $b^y = x$

example: verify sin 60 = 2 sin 30 cos 30

$$\frac{\sqrt{3}}{2} = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}}{2}$$

Now answer the following :

31 verify $\tan 60 = \dfrac{2\tan 30}{1-\tan^2 30}$
32. verify $\cos 120 = \cos^2 60 - \sin^2 60$
33. verify $\sin^2 45 + \cos^2 45 = 1$
34. Find $\log_2 32$
35. Solve for x: $\log x^2 = 4$

Polynomials

Your final topic for review involves working with polynomials. It is important that you are especially proficient at multiplying and dividing polynomials.
Try these:

36. $x^2(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 3x^{-1})$
37. $y^{\frac{1}{2}}(y^{\frac{5}{2}} - 3y^{\frac{3}{2}} + 4y^{\frac{1}{2}} - 8)$

38. $\dfrac{x^2 + 4x - 7}{x^{\frac{1}{2}}}$

39. $\dfrac{4x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 6x^{\frac{-1}{2}}}{2x^{\frac{-3}{2}}}$

40. $x^8\ (x^3 + 3x^2 - 4x + 5 + 2x^{-1} - 3x^{\frac{-4}{7}}$

<u>Answer Key</u>

1. 1 7 21 35 35 21 7 1

2. $r^4 + 4r^3s + 6r^2s^2 + 4rs^3 + s^4$

3. $9x^2 + 2(3)(4)xy + 16y^2$

4. $a^3 + 6a^2b + 12ab^2 + 8b^3$

5. $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$

6. Domain = $(-\infty, \infty)$ or all Real #'s ; Range = $(-\infty, \infty)$

7. Domain = $(-\infty, \infty)$; Range = $[-12.25, \propto)$

8. Domain = $(-\propto, 5) \cup (5, \propto)$

9. Domain = $[\frac{-5}{3}, \propto)$; Range = $[0, \propto)$

10. Domain = $(-\propto, -2] \cup [2, \propto)$; Range = $[0, 1) \cup (1, \propto)$

11. Domain = $(-\propto, -3) \cup (-3, 3) \cup (3, \propto)$

12. $y = -2x + 1$

13. $y = \frac{3}{4} x$

14. $y = -6$

15. $y = -x$

16. $y = \frac{2}{3}x - \frac{19}{3}$

17. $x = 6$

18. $y = 4x + 8$

19. $y = -2$

20. $(f + g)(x) = \sqrt{x} + \sin x$

21. $(f - g)(x) = \sqrt{x} - \sin x$

22. $(fg)(x) = \sqrt{x} \sin x$

23. $(f/g)(x) = \frac{\sqrt{x}}{\sin x}$, where $x \neq \frac{(2k+1)}{2}\pi$, $k \in Z$ (integers)

24. $f(g(x)) = \sqrt{\sin x}$

25. $(f + g)(x) = x^2 + 4x - 24$

26. $(f - g)(x) = x^2 + 4x - 18$

27. $(fg)(x) = -3x^2 - 12x + 63$

28. $(f/g)(x) = \dfrac{x^2 + 4x - 21}{-3}$

29. $f(g(x)) = -24$

30. $g(f(x)) = -4x^2 - 16x + 84$

31. $\sqrt{3} = \dfrac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \dfrac{3}{\sqrt{3}} = \sqrt{3}$

32. $\dfrac{-1}{2} = (\frac{1}{2})^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2$

$\dfrac{-1}{2} = \dfrac{-1}{2}$

33. $\left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2 = 1$

34. $\log_2 32 = y$
$2^y = 32$
$y = 5$

35. $x = 100$

36. $x^{\frac{7}{2}} + 2x^{\frac{5}{2}} - 3x$

37. $y^3 - 3y^2 + 4y - 8y^{\frac{1}{2}}$

38. $x^{\frac{3}{2}} + 4x^{\frac{1}{2}} - 7x^{\frac{-1}{2}}$

39. $2x^3 + x^2 - 3x$

40. $x^{11} + 3x^{10} - 4x^9 + 5x^8 + 2x^7 - 3x^{\frac{52}{7}}$