AP Calculus BC

Unit 1 – Limits and Continuity

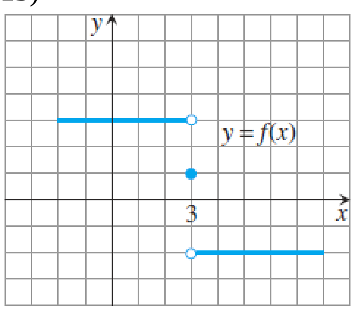
AP Calculus BC - Worksheet 1 **Limits**

Determine the limit. (Feel free to use L'Hopital's Rule)

1) $\lim\limits_{x\to c}\dfrac{x^4-x^3+1}{x^2+9}$	2) $\lim\limits_{x\to -4}(x+3)^{1998}$	3) $\lim\limits_{x\to 1}\left(x^3+3x^2-2x-17\right)$
4) $\lim\limits_{x\to \frac{1}{2}}(\operatorname{int} x)$	5) $\lim\limits_{x\to -2}\left(\dfrac{1}{x+2}\right)$	6) $\lim\limits_{x\to 0}\dfrac{(4+x)^2-16}{x}$
7) $\lim\limits_{x\to 0}\dfrac{5x^3+8x^2}{3x^4-16x^2}$	8) $\lim\limits_{x\to 0}\dfrac{\sin 2x}{x}$	9) $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x}$
10) $\lim\limits_{x\to 2}\dfrac{x+1}{x^2-4}$	11) $\lim\limits_{x\to 3}\dfrac{x^2-9}{x+3}$	12) $\lim\limits_{x\to 0^-}\dfrac{x}{\lvert x\rvert}$

For #13 and 14, use the graph of each function below to determine the indicated value.

13)



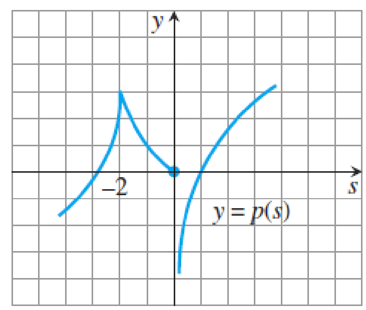
a) $\lim\limits_{x\to 3^-} f(x)$

b) $\lim\limits_{x\to 3^+} f(x)$

c) $\lim\limits_{x\to 3} f(x)$

d) $f(3)$

14)



a) $\lim\limits_{x\to -2^-} p(s)$

b) $\lim\limits_{x\to -2^+} p(s)$

c) $\lim\limits_{x\to -2} p(s)$

d) $p(-2)$

15)	Assume that $\lim\limits_{x\to 4} f(x)=0$ and $\lim\limits_{x\to 4} g(x)=3$. Determine each limit. a) $\lim\limits_{x\to 4}\left(g(x)+3\right)$ b) $\lim\limits_{x\to 4}\dfrac{g(x)}{f(x)-1}$ c) $\lim\limits_{x\to 4} g^2(x)$
16)	For $f(x)=\begin{cases}3-x, & x<2\\ \dfrac{x}{2}+1, & x>2\end{cases}$, evaluate $\lim\limits_{x\to 2} f(x)$.
17)	For $f(x)=\begin{cases}3-x, & x<2\\ 2, & x=2\\ \dfrac{x}{2}, & x>2\end{cases}$, evaluate $\lim\limits_{x\to 2} f(x)$.
18)	For $f(x)=\begin{cases}\sqrt{1-x^2}, & 0\le x<1\\ 1, & 1\le x<2\\ 2, & x>2\end{cases}$, determine the values of c for which $\lim\limits_{x\to c} f(x)$ exists.
19)	For $f(x)=\begin{cases}\sin x, & -2\pi\le x<0\\ \cos x, & 0\le x\le 2\pi\end{cases}$, at what values of c does $\lim\limits_{x\to c} f(x)$ exist?
20)	Find $\lim\limits_{x\to 0}\left(x\sin\dfrac{1}{x}\right)$ numerically (Graphing Calculator Permitted).

AP Calculus BC – Worksheet 2 **Limits involving Infinity**

For #1-2, find **(a)** $\lim_{x\to\infty} f(x)$ and **(b)** $\lim_{x\to-\infty} f(x)$. **(c)** Identify any horizontal asymptotes.

1) $f(x)=\dfrac{3x^3-x+1}{x+3}$	2) $f(x)=\dfrac{e^x}{x}$

For #3-4, Evaluate each limit.

3) $\lim_{x\to2^+}\dfrac{1}{x-2}$	4) $\lim_{x\to-3^+}\dfrac{x}{x+3}$
5) $\lim_{x\to\infty}\dfrac{3-9x+\sin 4x}{9x+\cos 4x}$	6) $\lim_{x\to-\infty}\left(5xe^{2x}\right)$

For #7-8, **(a)** find any vertical asymptotes of the graph of $f(x)$. **(b)** Describe the behavior of $f(x)$ to the left and right of each vertical asymptote.

7) $f(x)=\dfrac{x+3}{x-2}$	8) $f(x)=\dfrac{-2}{x^2-25}$

For #9-10 , describe the end behavior of the graph of $f(x)$.

9) $f(x)=\dfrac{x-2}{2x^2+3x-5}$	10) $f(x)=\dfrac{3x^2-x+5}{x^2-4}$

11)	Sketch a graph of a function, $f(x)$ that satisfies all of the stated conditions: $\lim_{x\to-\infty} f(x)=0$ $\lim_{x\to1^-} f(x)=4$ $\lim_{x\to1^+} f(x)=-2$ $f(1)=0$ $\lim_{x\to4^-} f(x)=-\infty$ $\lim_{x\to4^+} f(x)=\infty$ $\lim_{x\to\infty} f(x)=2$

AP Calculus BC - Worksheet 3 **Continuity**

<u>Show</u> (THREE STEPS) that each of the following functions is either continuous or discontinuous at the given value of x.

1. $f(x)=x+5$ at $x=1$	2. $f(x)=\dfrac{3x-1}{2x+6}$ at $x=-3$
3. $f(x)=\dfrac{x^2-16}{x-4}$ at $x=4$	4. $f(x)=\dfrac{x^2-25}{x+5}$ at $x=5$

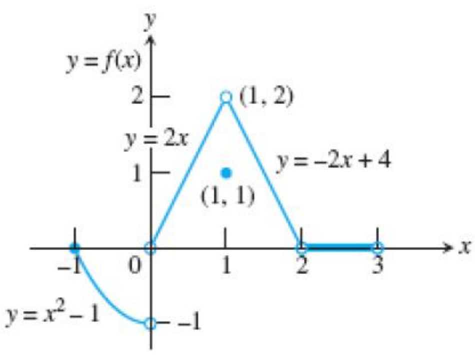
Give the open interval(s) on which the function is continuous.

5. $f(x)=x^2+2$	6. $f(x)=\dfrac{1}{x}$
7. $f(x)=\dfrac{x^2+1}{x-1}$	8. $f(x)=\dfrac{3x-5}{2x^2-x-3}$

Each of the following has a removable discontinuity. Find an extended function that is continuous at this discontinuity.

9. $f(x)=\dfrac{x^2-4}{x-2}$	10. $f(x)=\dfrac{x^2-5x+6}{x-3}$

12. Given the graph of $f(x)$ below, answer the following questions:



a) Is $f(x)$ continuous at $x=-1$? Explain

b) Is $f(x)$ continuous at $x=1$? Explain

c) At what values of x is $f(x)$ continuous?

d) Is it possible to extend $f(x)$ to be continuous at $x=0$? Why why not?

1

State whether the function $f(x)=\begin{cases} x^2-2x+1, & x<-1 \\ x+2, & -1\le x\le 2 \\ 2^x, & x\ge 2 \end{cases}$ is continuous at the given *x*-values. Justify your answers.

a) $x=-1$

b) $x=2$

2

State whether the function $f(x)=\begin{cases} x-x^2, & x<1 \\ x, & x=1 \\ \ln x, & x>1 \end{cases}$ is continuous at $x=1$. Justify your answers.

3

State whether the function $f(x)=\begin{cases} \cos x, & x\le \frac{\pi}{2} \\ \tan x, & \frac{\pi}{2}<x<\pi \\ \sin x, & x\ge \pi \end{cases}$ is continuous at the given *x*-values. Justify your answers.

a) $x=\frac{\pi}{2}$

b) $x=\pi$

4

Find the value of *k* that makes $f(x)=\begin{cases} 3-x^2, & x\le 4 \\ x+k, & x>4 \end{cases}$ a continuous function.

5

For each function, identify the type of discontinuity and where it is located.

a) $f(x)=\frac{x}{x+1}$

b) $g(x)=\frac{x+2}{x^2-2x-8}$

c) $h(x)=\frac{x^2+2x-3}{x+3}$

d) $f(x)=\sec 2x$ for $0\le x\le 2\pi$

e) $f(x)=\begin{cases} x^2+3, & x\le -1 \\ 5x-2. & x>-1 \end{cases}$

6

The function f has the properties indicated in the table below. Which of the following must be true?

b	$\lim\limits_{n\to b} f(x)$	$\lim\limits_{n\to b^+} f(x)$	$f(b)$
1	-1	3	3
2	5	5	8
3	1	1	1

(A) f is continuous at $x=1$.

(B) f is continuous at $x=2$.

(C) f is continuous at $x=3$.

(D) None of the above.

1) Use the Intermediate Value Theorem to show that $f(x) = x^3 + x$ takes on the value 9 for some x in $[1,2]$.

2) Show that $g(t) = \dfrac{t}{t+1}$ takes on the value 0.499 for some t in $[0,1]$.

3) Show that $f(x) = x^3 + 2x + 1$ has a solution in the interval $[-1,0]$.

4) Selected values of a continuous function *f* are given in the table below. What is the fewest possible number of zeros of *f* in the interval $[0,5]$?

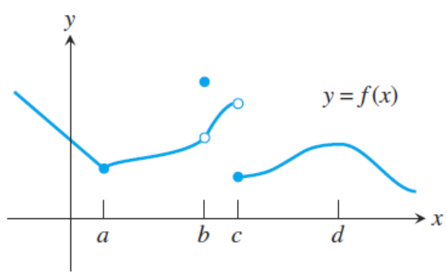
x	0	1	2	3	4	5
$f(x)$	1	-5	-4	2	-10	-15

Evaluate the limit if it exists.

5) $\lim_{x\to 4}\left(3+\sqrt{x}\right)$	6) $\lim_{x\to 1}\dfrac{5-x^2}{4x+7}$	7) $\lim_{x\to -1}\dfrac{3x^2+4x+1}{x+1}$
8) $\lim_{t\to 9}\dfrac{\sqrt{t}-3}{t-9}$	9) $\lim_{h\to 0}\dfrac{2(a+h)^2-2a^2}{h}$	10) $\lim_{x\to\infty}\dfrac{9x^2-4}{2x^2-x}$
11) $\lim_{x\to 3}\dfrac{x^2-4x-5}{x-3}$	12) $\lim_{x\to 0^-}\dfrac{\lvert x\rvert}{x}$	13) $\lim_{x\to 0}\dfrac{\sin x}{x+1}$

14) Describe the end behavior and identify any horizontal asymptotes on the graph of $f(x) = \dfrac{x^2-3x+2}{x^2-1}$.

15) Determine whether the indicated limit exists based on the graph below:



a) $\lim_{x\to b} f(x)$

b) $\lim_{x\to c^-} f(x)$

c) $\lim_{x\to d} f(x)$

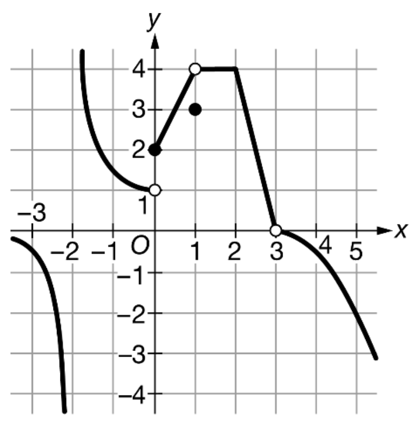
16) Determine if $f(x) = \begin{cases} \lvert x^3-4x\rvert, & x<1 \\ x^2-2x-2, & x\ge 1 \end{cases}$ is continuous at $x=1$.

17) Sketch a single graph of a function that satisfies all of the given conditions:

$\lim_{x\to\infty} f(x) = 3,\quad \lim_{x\to-\infty} f(x) = \infty,$

$\lim_{x\to 3^+} f(x) = \infty,\quad \lim_{x\to 3^-} f(x) = -\infty$

18) Determine if $f(x) = \dfrac{2x+1}{x^2-2x+1}$ has any discontinuities. State whether the discontinuities are removable, jump, or infinite.



x	0.8	0.9	0.09	0.009	1.001	1.01	1.1	1.2
$g(x)$	4.16	4.59	4.960	4.996	5.004	5.040	5.39	5.76

1) The graph of the function f is shown in the xy-plane above. The graph of f has a vertical asymptote at $x = -2$. The function g is continuous and increasing for all x. Values of $g(x)$ at selected values of x are shown in the table above.

a) Using the graph of f and the table for g, estimate $\lim_{x \to 1}(2f(x) + 3g(x))$.

b) For each of the values $a = -2$, $a = 2$, and $a = 3$, determine whether or not f is continuous at $x = a$. In each case, the three-part definition of continuity to justify your answer.

c) Find the value of $\lim_{x \to 0} f(f(x))$ or explain why the limit does not exist.

2) The function, $Y(t)$, is a piecewise-defined function defined by:

$$Y(t) = \begin{cases} 10e^{0.05t} & \text{for } 0 \le t \le 10 \\ f(t) & \text{for } 10 < t \le 12 \\ \dfrac{600}{20 + 10e^{-0.05(t-12)}} & \text{for } t > 12 \end{cases},$$

where $f(t)$ is a continuous function such that $f(12) = 20$.

a) Find $\lim_{t \to \infty} Y(t)$.

b) Is the function $Y(t)$ continuous at $t = 12$. Justify your answer.

c) The function Y is continuous at $t = 10$. Is there a time t, for $0 < t < 12$, at which $Y(t) = 18$. Justify your answer.