Black Horse Pike Regional School District Curriculum Template

ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21ST CENTURY GLOBAL SKILLS

Course Name: AP Calculus AB

Course Number: 0343000

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Course/Unit Title: AP Calculus AB **Grade Level(s):** **11-12**	**Unit Summary**: In this unit students develop an understanding of limits as the foundational building blocks for both derivatives and integration. It is essential for discovering and developing important ideas, definitions, formulas and theorems in calculus. Students will solve limit problems graphically, algebraically, and conceptually. They will generate and work with tables, sketch and analyze various graphs, and apply numerous algebraic techniques to find limits of indeterminate forms. Students must have a solid, intuitive understanding of limits and be able to compute various limits, such as, one-sided limits, limits at infinity, infinite limits, and trigonometric limits. In addition, they will communicate both orally and in written form effectively what their answers mean in the context of the problems they are given. Finally, students will understand how limits are used to determine continuity, which is a fundamental property of functions, and apply the Intermediate Value Theorem.
Essential Question(s): **How can the concept of a limit be used to understand the behavior of functions?** **How can a function have a limit at certain *x* values even though the function is undefined at that *x* value?** **How do I determine the value of a limit graphically, algebraically and conceptually?** **What is continuity?** **How are limits used to define continuity?**	**Enduring Understanding(s):** Students will be able to: • Understand that the tangent line problem is basic to calculus. • Estimate limits of functions • Estimate a limit using a numerical or graphical approach. • Interpret limits expressed symbolically. • Learn different ways a limit can fail to exist. • Express limits symbolically using correct notation. • Determine limits of functions using properties of limits and algebraic manipulation. • Determine the limit of trigonometric functions and using the squeeze theorem. • Analyze functions for intervals of continuity or points of discontinuity. • Determine one-sided limits. • Use properties of continuity. • Determine the applicability of the Intermediate Value Theorem using continuity. • Deduce and interpret behavior of functions using limits. • Determine infinite limits from the left and from the right. • Find and sketch vertical asymptotes of a function.

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

Applications of the Mathematical Practices for AP Calculus:

Learning Target

MPAC 1 – Reasoning with definitions and theorems:

Students discuss real-world problems where the Intermediate Value Theorem (IVT) is applicable, such as using the velocity of a commercial airline. On the runway before leaving, the velocity of the plane is zero (not moving). Most commercial airlines will reach their maximum velocity of about 500 miles per hour at cruising altitude. Students are able to discuss whether this is a continuous function and if so, at some point does the plane have to be flying at 300 miles per hour? This activity has students practice applying the IVT in real-world applications.

MPAC 2 – Connecting concepts:

Students work through a handout containing several limit problems in which *x* goes to infinity. As they work through the problems students come up with a strategy for quickly solving this type of limit problem by focusing on the term with the largest exponent in the numerator and denominator. This allows students to visualize how a function can approach a certain *y* value but never actually reach this value.

MPAC 3 – Implementing algebraic/computational processes:

Throughout this unit students realize the importance of algebraic procedures, such as factoring and rationalizing numerators/denominators to help solve limit problems.

MPAC 4 – Connecting multiple representations:

Throughout this unit students solve problems using a graphical approach, a numerical approach, an algebraic approach and a verbal or written approach, communicating what their final answer means in the context of the problem. They are able to see how different approaches help to solve different types of problems.

MPAC 5 – Building notational fluency:

Throughout this unit students learn how to apply and use limit notation, and how to use this notation to help solve problems.

MPAC 6 – Communicating:

Students learn how to clearly interpret limit notation and communicate their results into meaningful answers.

Inter-Disciplinary Connections:

Real World and Inter-disciplinary problems:

Larson: pg. 55: Modeling the cost of a phone call between two cities; pg. 67: Free-falling objects velocity and position; pg. 79: Telephone charges, Inventory Management, Volume; pg. 86: theory of relativity, rate of change, cost, Boyle's Law.

Foerster: Section 1-1: Change in angle of an automatic door, Pendulum problem, Cost of a board; Section 2-2: Distance from a traffic light when it changes to green; Section 2-4: Pizza Delivery Problem; Section 2-4: River Crossing Problem; Section 2-6: Foot race problem, Postage stamp problem; Section 2-7: Projectile motion, Determine how far a glacier has traveled in a given interval of time, Temperature vs Depth,

Students will engage with the following text, resources and tools:

Texts:

- Calculus of a Single Variable, Seventh Edition - By Larson, Hostetler and Edwards
- Calculus Concepts and Applications – by Paul A. Foerster

Online Resources incorporated through the year, include but not limited to:

- Albert io – online AP exam practice resource
- AP Central - Previously published and released AP questions
- Desmos – online graphing tool

Calculators:

- TI – 84 Plus
- TI - 89 Titanium

The following 21st century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments and projects:

21st Century skills:	**Mathematical Practices:**
• Critical thinking • Creativity • Collaboration • Communication • Information literacy • Technology literacy • Media literacy • Flexibility • Leadership • Initiative • Productivity • Social skills	• Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics • Use appropriate tools strategically • Attend to precision • Look for and make use of structure • Look for and express regularity in repeated reasoning

Students will write:

What the domain of a function is given it algebraically.

What possible errors can be made by determining the domain of a function solely by analyzing a function's graph.

The importance of examining a function analytically as well as graphically.

A brief description of the meaning to the notation $\lim_{x \to c} f(x)$

Compare $f(x) = x$, $g(x) = sinx$, and $h(x) = \frac{sinx}{x}$ graphically and write why $\lim_{x \to 0} h(x) = 1$.

Compare $f(x) = x$, $g(x) = \sin^2 x$, and $h(x) = \frac{\sin^2 x}{x}$ graphically and write why $\lim_{x \to 0} h(x) = 0$.

About the importance of examining a function analytically as well as graphically when determining continuity.

Descriptions about how functions differ.

Meanings of different types of discontinuity and explain why.

Explanations as to why the intermediate value theorem applies on a given closed interval.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills.

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

Section:1.1 – A Preview of Calculus

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Having students write the equation of a line given two points.
Teaching Objectives	• Understand that the tangent line problem is basic to calculus.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg.47 Foerster Text*:* pg. 5

Section:1.2 – Finding Limits Graphically and Numerically

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Having students find the average rate of change of a function given an interval.
Teaching Objectives	• Estimate limits of functions • Estimate a limit using a numerical or graphical approach. • Interpret limits expressed symbolically. • Learn different ways a limit can fail to exist. • Express limits symbolically using correct notation.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 54 – 56 Foerster Text*:* pg. 33 & 37 – 39

Section:1.3 – Evaluating Limits Analytically

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students evaluate all six trig functions at given values on the unit circle.
Teaching Objectives	• Determine limits of functions using properties of limits and algebraic manipulation. • Determine the limit of trigonometric functions and using the squeeze theorem.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 65 – 67 Foerster Text*:* pg. 43 – 45

Section:1.4 – Continuity and One-Sided Limits

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Give students a graph with points of discontinuity and have them write down where they think the graph is continuous.
Teaching Objectives	• Analyze functions for intervals of continuity or points of discontinuity. • Determine one-sided limits. • Use properties of continuity. • Determine the applicability of the Intermediate Value Theorem using continuity.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 76 – 79 Foerster Text*:* pg. 49 – 52 & 61 – 63

Section:1.5 – Infinite Limits

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students determine asymptotes of rational functions.
Teaching Objectives	• Deduce and interpret behavior of functions using limits. • Determine infinite limits from the left and from the right. • Find and sketch vertical asymptotes of a function.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 85 – 87 & Chapter review: pg. 88 – 91 Foerster Text*:* pg. 57 – 59 & Chapter review: pg. 64 – 67

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.
IDENTIFY BLOOM'S LEVELS.

Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

Accommodations/Modifications:

As per individual student's IEP or 504 plan.

Summative Assessments:

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- **Diagnostic Pre-Test**
- **Chapter Tests**
- **Projects**
- **End-Of –Course Assessment**

Accommodations/Modifications:

As per individual students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- **Projects**
- **Practice AP Exam Questions**
- **Homework**
- **Classwork**

Accommodations/Modifications:

As per individual student's IEP or 504 plan.

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ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21ST CENTURY GLOBAL SKILLS

Course Name: AP Calculus AB

Course Number: 0343000

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Course/Unit Title: AP Calculus AB **Grade Level(s):** **11th and 12th**	**Unit Summary**: In this unit students will get into the heart of calculus. Derivatives are a major portion of the course, so they will spend a significant amount of time in this unit. Derivatives are used to describe the rate of change of one variable with respect to another variable to understand change in a variety of contexts. At first students build the derivative using the concept of limits and use it primarily to compute the instantaneous rate of change of a function. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. As they progress through this unit they will spend some time on the relationship between position, velocity and acceleration on problems involving projectile motion and rectilinear motion. Students will go on to understand and apply the Mean Value Theorem and will have the opportunity to see how the average rate of change can be used to justify and instantaneous speed. They will also spend a significant amount of time developing a comprehensive analysis of functions (for example, determining whether a function is increasing or decreasing and finding concavity and extreme values), using not only their graphs but their derivatives as well. Students should be able to solve separable differential equations, and be familiar with a variety of real-world applications, including related rates, optimization and growth and decay models. This is most likely the first time students will be asked to think deeply on a conceptual level, so they may struggle to make connections at first. Students often ask how far they should simplify their answers and it should be stressed that they should look to simplify only if it provides meaningful progress, such as a much shorter or cleaner answer. However, it is important that students are able to identify alternative forms of their answers on multiple-choice questions on the AP exam.
Essential Question(s): **How can we use the tangent line to get the slope of a function at a specific *x* value?** **How is the derivative of a function defined as the limit of a difference quotient?** **What is differentiability?**	**Enduring Understanding(s):** Students will be able to: • Identify the derivative of a function as the limit of a difference quotient. • Solve problems involving the slope of a tangent line. • Use the limit definition to find the derivative of a function. • Recognize the connection between differentiability and continuity. • Estimate derivatives. • Calculate derivatives. • Interpret the meaning of a derivative within a problem. • Find the derivative of a function using the following rules: Constant Rule, Power Rule, Constant Multiple Rule, and the Sum and Difference Rules.

How can a derivative be determined using a variety of strategies? **How can we expand on our understanding of derivatives using equations that are not mathematical functions?** **How can a derivative of a function be used to understand the behavior of the given function?** **How can a derivative be interpreted and applied in situations that involve instantaneous rate of change?** **What do the first and second derivatives of a function tell us about its behavior?** **How does the Mean Value Theorem connect the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval?** **How can derivatives be used to solve optimization problems?**	• Find the derivative of the sine function and cosine function. • Use derivative to find rates of change. • Solve problems involving rectilinear motion. • Solve problems involving projectile motion. • Find the derivative using the Product Rule and Quotient rule. • Find the derivative of the tangent, cotangent, secant and cosecant functions. • Determine higher-order derivatives. • Find the derivative of functions using the Chain Rule. • Find derivatives of functions involving the natural logarithmic function • Differentiate natural exponential functions. • Differentiate exponential functions that have bases other than e • Differentiate logarithmic functions for bases other than e. • Recognize limits that produce indeterminate forms. • Apply L'Hospital's Rule to evaluate a limit. • Distinguish between functions written in implicit form and explicit form. • Use implicit differentiation to find the derivative of a function. • Solve problems involving related rates • Solve problems involving rates of change in applied contexts. • Use related rates to solve real-life problems. • Use derivatives to analyze properties of a function • Find extrema on a closed interval. • Apply the Mean Value Theorem to describe the behavior of a function over an interval. • Understand and use Rolle's Theorem. • Determine intervals on which a function is increasing or decreasing. • Apply the First Derivative Test to find relative extrema of a function. • Determine intervals on which a function is concave upward or concave downward. • Find points of inflection for a function. • Apply the Second Derivative Test to find relative extrema of a function. • Determine limits at infinity. • Determine the horizontal asymptotes of a function. • Determine infinite limits at infinity. • Analyze and sketch the graph of a function. • Solve problems involving optimization. • Approximate a zero of a function using Newton's Method • Understand the concept of a tangent line approximation • Compare the value of the differential dy, with the actual change in y • Estimate a propagated error using a differential • Find the differential of a function using differentiation formulas

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

Applications of the Mathematical Practices for AP Calculus:

Learning Target

MPAC 1 – Reasoning with definitions and theorems:

Students the graph $y = sinx$ over a very small interval is displayed so that it appears to be a line. After class discussion about the graph, the graph $y = sinx$, without a restricted domain, is projected. This allows for discussion about the concept of local linearity and can be applied to tangent lines. This is used to talk about how the slope of the tangent line is equal to the slope of the function at specific *x*-values. This observation and discussion allows students to work toward proficiency with the limit definition of the derivative.

Using the operator $\frac{d}{dx}$ on a function and the notation $\frac{dy}{dx}$ for derivative helps students understand the definition of implicit differentiation. They use this reasoning to help understand this new type of derivative.

Combining the four theorems (the Intermediate Value Theorem, Extreme Value Theorem, Roll's Theorem and the Mean Value Theorem) that students need to know into one activity allows students to develop an understanding of how important they are for solving problems and justifying their answers.

MPAC 2 – Connecting concepts:

Using graphing calculators to explore tangent line(s) of various functions to estimate the slope of a curve at specific *x* values provides students with the understanding that limits are used to calculate derivatives, and allows them to recognize the importance of the tangent line and local linearity.

Having students sketch a picture to represent the situations presented in related rates problems, provides visual representations and allows discussion to occur about what variables are changing with respect to time and what is being asked to solve for.

Have students take the first and second derivatives of a polynomial function such as $f(x) = x^4 - 4x^3$ and set each of these equal to zero finding all critical values. Apply the first derivative test and concavity test to determine what these values represent (max, min or points of inflection) and then substituting them into the original function to find the y-component of the coordinate. This allows students to understand how to analyze a function's behavior and how to accurately sketch these functions without the use of a graphing calculator.

MPAC 3 – Implementing algebraic/computational processes:

To introduce the product rule give the students $y = (x^2)(x^3)$. At this point students can quickly take the derivative of the function by first simplifying the equation to $y = x^5$ and then using power rule to determine $y' = 5x^4$. However, explore the original problem given by taking the derivative using the product rule which in the end produces the same answer. Similarly introduce the quotient rule using $y = \frac{x^7}{x^2}$ which, will again provide the derivative $y' = 5x^4$. Afterwards, encourage discussion of using the product rule on this given problem by making the exponent of the denominator negative. Finally discuss how sometimes the best approach to taking derivatives is easier to first simplify the problems such as $y = \frac{3x^2-2x+1}{x^4}$ and then use power rule or sometimes you have no choice but to use product rule and quotient rules in problems such as $y = x^4 sinx$ & $y = \frac{3x^2-2x+1}{x^4+1}$.

To find derivatives implicitly students need to be able to use numerous algebraic and computational processes correctly.

To sketch a curve accurately students must algebraically manipulate the given equation to determine characteristics such as domain, intercepts, asymptotes, and holes. They must then find the first two derivatives of the function and analyze and interpret their critical values to provide maximums, minimums, points of inflection, intervals of increase/decrease and intervals of concavity. Students gain an understanding of how important derivatives are for analyzing and understanding how functions behave.

MPAC 4 – Connecting multiple representations:

Students are able to estimate/calculate derivatives from equations/graphs/tables and are able to correctly interpret both first and second derivative graphs.

With implicit differentiation students are able to gain an understanding that all equations (whether it's a function or not) have derivatives and they are able to calculate them precisely.

While curve-sketching students can connect the algebraic component of derivatives with the graphical interpretation of the function and first and second derivatives graphs.

MPAC 5 – Building notational fluency:
Students are able to understand the variety of notations for derivatives and higher-order derivatives. They understand that the derivative on position produces velocity and the derivative of velocity yields acceleration.

Students should be able to connect appropriate notation to different representations and interpret this notation in a contextual setting. This is strongly emphasized in related rate problems since they are all real world applications of derivatives.

Students should be able to make connections to a given function, its first derivative and second derivative and what graphical characteristics each one yields about the given function.

MPAC 6 – Communicating:
Students practice their writing to clearly explain and justify their answers in the context of a problem and how this information allows us to make decisions contextually.

Related rate problems all students the opportunity to explain the connection between the concepts of rate of change and how variables are changing with respect to time.

Students are able to make connections, apply, discuss and justify their finial answers for problems that involve the Intermediate Value Theorem, Extreme Value Theorem, Roll's Theorem and the Mean Value Theorem.

Inter-Disciplinary Connections:

Real World and Inter-disciplinary problems:
Larson: Section 2-2: Vertical motion, Volume rate of change, Area rate of change, Stopping distance of a vehicle, Inventory management, Fuel Cost, Newton's Law of Cooling; Section 2-3: Finding acceleration due to gravity, Population growth, Inventory replenishment, Boyle's Law, Position, velocity and acceleration; Section 2-4: Doppler effect, Harmonic Motion, Pendulum, Wave motion, Circulatory System, Cost of producing a product; Section 2-5: Orthogonal Trajectories; Section 2-6: Change of Volume as a balloon is being inflated, Speed of an airplane tracked by radar, Change in angle of elevation as a rocket is launched, Velocity of a piston, Change in area as a dimension increases, Change surface area of a cube as a dimension increases, Change of depth in a conical tank as it is being filled, Change in height of a ladder as the bottom is being pulled away, Machine design, Change of evaporation, Change in shadow length, Linear vs Angular speed, Refrigeration; Section 3-1: Power, Lawn Sprinkler angle, Inventory Cost, Highway design; Section 3-2: Vertica motion, Reorder cost, Sales; Section 3-3: Path of a projectile, Trachea contraction, Profit, Power, Electrical Resistance; Section 3-4: Aircraft glide path, Highway design, Beam deflection, Average cost, Inventory cost; Section 3-5: Average cost, Engine efficiency, Average temperature, Average typing speed; Section 3-6: Number of bacteria in a culture; Section 3-7:

Dimensions of a box with a given surface area that yields maximum volume, Position of a stake to be placed between two different poles that uses the least amount of wire, Dimensions of a rectangle that yield a maximum area given a set perimeter, Traffic control, Chemical reaction, Maximum Profit, Beam strength, Projectile range, Illumination, Minimum time, Minimum cost, Minimum Force; Section 5-1: Home mortgage, Sound intensity, Atmospheric pressure; Section 5-4: Depreciation, Harmonic motion, Depreciation; Section 5-5: Inflation, Depreciation, Compound interest, Population growth.

Foerster: Section 3-3: Dimension tolerance; Section 3-4: Position and velocity; Section 3-5: Projectile motion, speeding up vs slowing down, Displacement of a particle, velocity and its acceleration, Car problem, Sky diver's acceleration, Clock pendulum acceleration from velocity, Average velocity vs instantaneous velocity, Compound interest, Radioactive decay; Section 3-7: Beanstalk problem, Balloon volume; Section 3-8: Harmonic motion, Ferris wheel, Pendulum problem, Daylight problem; Section 3-9: Compound interest, Radioactive tracer, Altimeter problem; Chap 3 review: Leaky tire, Spaceship problem, Shark weight problem, Clock problem, Biological half-life; Section 4-2: Bouncing spring problem, Tacoma Narrows bridge problem, Pole dance problem; Section 4-3: Black hole problem, Catch-up rate; Section 4-4: Light on the monument problem, Point of light problem; Section 4-5: Radar problem, Exit sight problem; Section 4-6: Railroad curve problem, Bicycle frame design, Baseball line drive; Section 4-9: Bacteria spreading, Change of Volume in a balloon being inflated, Kinetic energy, Base Runner problem, Rate of change in distance as a tugboat moves, Rate of change in water depth as enclosed area gets smaller, Rate of change in distance between two ships as they travel, Rate of change in diameter of spotlight as it moves closer to the surface, Planetary motion; Section 8-3: Divided stock pen and maximum area for given amount of fence length, Motel problem, Two corral problem, Open box and maximum volume, Shortest distance, Track and field problem, Ladder problem, Tin can problem, Cup problem, Duct Problem, Conical missile nose cone problem.

Students will engage with the following text, resources and tools:

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Online Resources incorporated through the year, include but not limited to:

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- AP Central - Previously published and released AP questions
- Desmos – online graphing tool

Calculators:

- TI – 84 Plus
- TI - 89 Titanium

The following 21st century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments and projects:

21st Century skills:

- Critical thinking
- Creativity
- Collaboration
- Communication
- Information literacy
- Technology literacy
- Media literacy
- Flexibility
- Leadership
- Initiative
- Productivity
- Social skills

Mathematical Practices:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

Students will write:

Descriptions of the geometric significance of differentiability of two functions.
The meaning of $f'(1.49)$ given $N = f(p)$ is the function that yields the number of gallons of gasoline sold by a station at a price of dollars per gallon.
The criteria used in selecting a graph as f and f' when given two graphs.
Justifications as to why or why not a function has a tangent line at a given point.
The criteria for a function to have a horizontal tangent present.
Connections to differentiability and continuity.
Appropriate units with all answers that model real world situations – On the AP exam students may not earn a point on a free-response question when unit are not included with the solution.
Explanations how the velocity of an object can be determined given a graph of its position.
Describe the difference between the explicit form of a function and an implicit equation.
The difference between a negative rate of change and a positive rate of change.
Explain if x changes at a constant rate, does y change at a constant rate and if so is it the same a x given $y = ax + b$
Explain and justify their approach for solving related-rate problems.
How derivatives can be used to approximate the zero of a function.
The difference between relative and absolute extrema.
The graph of a function on a closed interval given extrema locations.
Explain why or why not the Mean Value Theorem applies to a function on a closed interval.
Explain how the Mean Value Theorem and Rolle's theorem are similar and different.
How you can determine the intervals on which a function is increasing or decreasing.
How you can determine the location of a function's relative extrema using derivatives.
How to determine the concavity of a function using derivatives.
How can derivatives be used to sketch the graph of a function precisely.
Justify the possibility of a function crossing its horizontal or vertical asymptotes.
To explain since the surface area of a bottle does not change when squeezed how does the volume change.
Justify how to maximize or minimize quantities.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills.

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

Section: 2.1 – The Derivative and the Tangent Line Problem

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Having students find the equation of a that goes through a given point and is parallel to another line.
Teaching Objectives	• Identify the derivative of a function as the limit of a difference quotient. • Solve problems involving the slope of a tangent line. • Use the limit definition to find the derivative of a function. • Recognize the connection between differentiability and continuity. • Estimate derivatives.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 101 – 104 Foerster Text*:* pg. 73, 76 – 78, 81 – 84

Section: 2.2 – Basic Differentiation rules and Rates of Change

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Having students find the derivative of a polynomial function using the definition of derivative
Teaching Objectives	• Calculate derivatives. • Interpret the meaning of a derivative within a problem. • Find the derivative of a function using the following rules: Constant Rule, Power Rule, Constant Multiple Rule, and the Sum and Difference Rules. • Find the derivative of the sine function and cosine function. • Use derivative to find rates of change. • Solve problems involving rectilinear motion. • Solve problems involving projectile motion.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 113 – 116 Foerster Text*:* pg. 90 – 92, 97 – 100, 143 - 145

Section: 2.3 – The Product and Quotient Rules and Higher Order Derivatives

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Having students find the equation derivative of the product of two binomials by expanding the expression and using the power rule.
Teaching Objectives	• Calculate derivatives • Find the derivative using the Product Rule and Quotient rule. • Find the derivative of the tangent, cotangent, secant and cosecant functions. • Determine higher-order derivatives.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 124 - 126 Foerster Text*:* pg. 131, 134 – 137, 139 – 142

Section: 2.4 – The Chain Rule

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Having students find the derivative of a quantity squared by first expanding and then using power rule
Teaching Objectives	• Calculate derivatives • Find the derivative of functions using the Chain Rule.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 133 - 136 Foerster Text*:* pg. 105 – 106

Section: 5.1 – The Natural Logarithmic Function: Differentiation

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Expand a logarithmic expression
Teaching Objectives	• Find derivatives of functions involving the natural logarithmic function
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 321 – 232 Foerster Text*:* pg. 120 - 122

Section: 5.4 – Exponential Functions: Differentiation and Integration

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Find the derivative of a polynomial, rational, trigonometric and logarithmic expression
Teaching Objectives	• Differentiate natural exponential functions
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 347 – 350 *Only problems involving derivatives Foerster Text*:* pg. 120 - 122

Section: 5.5 – Bases Other than e and Applications

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Find the derivative of a polynomial, rational, trigonometric and logarithmic expression
Teaching Objectives	• Differentiate exponential functions that have bases other than *e* • Differentiate logarithmic functions for bases other than *e*.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 357 – 360 *Only problems involving derivatives Foerster Text*:* pg. 120 - 122

Section: 7.7 – Indeterminate Forms and L'Hopital's Rule

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Find the limit of a rational function with an asymptote
Teaching Objectives	• Recognize limits that produce indeterminate forms. • Apply L'Hospital's Rule to evaluate a limit.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 537 – 539 Foerster Text*:* pg. 299 – 301

Section: 2.5 – Implicit Differentiation

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Discuss what makes an equation a function.
Teaching Objectives	• Distinguish between functions written in implicit form and explicit form. • Use implicit differentiation to find the derivative of a function. • Solve problems involving related rates • Solve problems involving rates of change in applied contexts.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 142 – 143 Foerster Text*:* pg. 172 - 174

Section: 2.6 – Related Rates

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Find the derivative of a position function and discuss its meaning, units and notation.
Teaching Objectives	• Use related rates to solve real-life problems.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 149 - 152 Foerster Text*:* pg. 176 – 179

Section: 3.1 – Extrema on an Interval

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students identify locations of a maximum using the feature on the calculator
Teaching Objectives	• Use derivatives to analyze properties of a function • Find extrema on a closed interval.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 165 – 167 Foerster Text*:* pg. 371, 380 - 385

Section: 3.2 – Rolle's theorem and the Mean Value Theorem

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have determine where a function has horizontal tangents.
Teaching Objectives	• Apply the Mean Value Theorem to describe the behavior of a function over an interval. • Understand and use Rolle's Theorem.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 172 - 173 Foerster Text*:* pg. 217 - 221

Section: 3.3 – Increasing and Decreasing Functions and the First Derivative Test

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students identify intervals of increase and decrease by viewing a graph.
Teaching Objectives	• Use derivatives to analyze properties of a function • Determine intervals on which a function is increasing or decreasing. • Apply the First Derivative Test to find relative extrema of a function.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 181 - 183 Foerster Text*:* pg. 380 – 385

Section: 3.4 – Concavity and the Second Derivative Test

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students find the first two derivatives of a rational function.
Teaching Objectives	• Use derivatives to analyze properties of a function • Determine intervals on which a function is concave upward or concave downward. • Find points of inflection for a function. • Apply the Second Derivative Test to find relative extrema of a function.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 189 - 191 Foerster Text*:* pg. 380 – 385

Section: 3.5 – Limits at Infinity

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students determine where a rational function has holes, vertical asymptotes and horizontal asymptotes.
Teaching Objectives	• Use derivatives to analyze properties of a function • Determine limits at infinity. • Determine the horizontal asymptotes of a function. • Determine infinite limits at infinity.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 199 – 201 Foerster Text*:* pg. 57 – 59

Section: 3.6 – A Summary of Curve Sketching

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students list all information needed to sketch a polynomial function precisely without a calculator
Teaching Objectives	• Use derivatives to analyze properties of a function • Analyze and sketch the graph of a function.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 208 - 210 Foerster Text*:* pg. 380 - 385

Section: 3.7 – Optimization Problems

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students list possible dimensions for a rectangular yard given the area.
Teaching Objectives	• Use derivatives to analyze properties of a function • Analyze and sketch the graph of a function.
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 216 - 221 Foerster Text*:* pg. 387 - 395

Section: 3.8 – Newton's Method

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students find the equation of a line tangent to a function at a given x coordinate
Teaching Objectives	• Approximate a zero of a function using Newton's Method
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 226 - 227 Foerster Text*:* pg. 182 - 183

Section: 3.9 - Differentials

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but not limited to:* Have students list possible dimensions for a rectangular yard given the area.
Teaching Objectives	• Understand the concept of a tangent line approximation • Compare the value of the differential dy, with the actual change in y • Estimate a propagated error using a differential • Find the differential of a function using differentiation formulas
Checking for Understanding	*Suggestions include but not limited to:* Exit Tickets (teacher made supplement) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text pg. 233 - 237 Foerster Text*:* pg. 194 - 196

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.
IDENTIFY BLOOM'S LEVELS.

Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

Accommodations/Modifications:

As per individual student's IEP or 504 plan.

Summative Assessments:

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- **Diagnostic Pre-Test**
- **Chapter Tests**
- **Projects**
- **End-Of –Course Assessment**

Accommodations/Modifications:

As per individual students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- **Projects**
- **Practice AP Exam Questions**
- **Homework**
- **Classwork**

Accommodations/Modifications:

As per individual student's IEP or 504 plan.

Black Horse Pike Regional School District Curriculum Template

ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21ST CENTURY GLOBAL SKILLS

Course Name: AP Calculus AB

Course Number: 0343000

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Course/Unit Title: AP Calculus AB **Grade Level(s):** **11-12**	**Unit Summary**: Students will discover the relationship between differentiation and integration as inverse operations. Students will learn how to integrate functions using definite integrals and indefinite integrals. Students will use integration to solve many real-world applications. Students will also learn the importance of the Fundamental Theorem of Calculus and its many applications. Students will revisit differentiation when they learn to derive and integrate using transcendental functions. This will instill the importance of the chain rule which they learned in the Derivative unit. Students will also learn to interpret and sketch slope fields, and solve differential equations. Students also will use integration to find the area between two curves. Finally, student will use rotation and other techniques to find the volume of a variety of 3-D solids as well as of solids with known cross-sectional areas.
Essential Question(s): **How can we see that antidifferentiation is the Inverse process of differentiation?** **How can we show that the definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies?** **How does the fundamental theorem of calculus, which has two distinct formulations, connect differentiation and integration?** **How can we use the definite integral of a function over an interval as a mathematical tool**	**Enduring Understanding(s):** Students will be able to: • Recognize antiderivatives of basic functions. • Interpret the definite integral as the limit of a Riemann sum. • Express the limit of a Riemann sum in integral notation. • Approximate a definite integral • Calculate a definite integral using areas and properties of definite integrals. • Analyze functions defined by an integral. • Calculate antiderivatives • Evaluate definite integrals. • Interpret the meaning of a definite integral within a problem • Apply definite integrals to problems involving motion. • Apply definite integrals to problems involving area and volume. • Use the definite integral to solve problems in various contexts. • Analyze differential equations to obtain general and specific solutions. • Interpret, create, and solve differential equations from problems in context.

with many interpretations and applications involving accumulation? **How can we use antidifferentiation to solve problems with separable differential equations?**	

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

Applications of the Mathematical Practices for AP Calculus:

Learning Target

MPAC 1 – Reasoning with definitions and theorems:

In all the activities through the unit students develop the ability to use the various FTC formulas to problem solve in a variety of different problems. Students need to be able to make conjectures on tasking derivatives of transcendental functions where the "rules" are different from polynomial functions. Students learn to build arguments about antiderivatives with *+C* creating a family of solutions rather than a single solution. Students will get to see how/why the formulas we use for area/volume work and how they are derived.

MPAC 2 – Connecting concepts:

Students will discover the derivative and integration and inverse operation. Students will quickly see during integration of *e^x* that transcendental functions behave differently from the polynomial functions they studied earlier. From the slope field activities students should be able to match both the associated differential equation and function connecting these two concepts.

MPAC 3 – Implementing algebraic/computational processes:

Students learn to select appropriate strategies and complete algebraic computations correctly (like in u-substitution). In both slope field and differential equation activities, students learn the importance of correctly using the rules of algebra to separate the variable and then integrating using good mathematics.

MPAC 4 – Connecting multiple representations:

In the Average Value of a Function, students can relate this new calculus formula to a very simple formula they studied in geometry, making the new formula easier to remember. Students will learn to visualize the "families" of vertically stacked functions and their connection to constant of integration *+C*. Students learn how to use the formulas for volume of a solid to help them visualize what the 3-D solid looks like.

MPAC 5 – Building notational fluency:

When Deriving the Evaluative Component of the Fundamental Theorem of Calculus students learn how important correct notation can help them understand problems and relationships between the first derivative and its corresponding functions. When defining *e* students will learn how to interpret various nomenclatures and to correctly differentiate and integrate the different transcendental functions. Students need to be comfortable using proper notation with differential equations and then solving for their final answer in the format *y=.* Students will make a further connection to the definite integral of a Riemann Sum when working on volume of a shape with a known cross section as well as when studying rotation of solids.

MPAC 6 – Communicating:

Students learn how to interpret the results from their graphing calculators for solving problems where answers need precision. Students need to explain the relationship between differential equations and the function, as well as communicating the results they get from a slope field. Students need to correctly use units in their final answers and explain their final answers in a contextual setting: i.e. for area problems units should be squared feet, and volume problems units should be feet cubed.

Inter-Disciplinary Connections:

Real World and Inter-disciplinary problems:

Larson: pg. 251: Express the height of a falling object as a function of its rate of change.; p. 285: Compute Force; Blood Flow; Respiratory Cycle; Average Profit Average Sales; Model various data for business.; p.299: Cash Flow Depreciation; Rainfall; Sales; Water Supply; Electricity.; p.350 Probability like finding mean waiting time for people waiting for service and average lifetime for a car battery.

Foerster: p.217-218: Compound Interest for a retirement account; Softball line drive by displacement; Turnpike Problem computing speed via point of entrance and time at which you entered.; P.237: Displacement of a car problems; Acceleration of vehicles; area problems; The amount of work needed to stretch equals the force exerted on the spring times the displacement of the end of the spring from one point to another; Heat problems. P. 247 Computing the volume of solids for manufacturing.

Students will engage with the following text, resources and tools:

Texts:

- Calculus of a Single Variable, Seventh Edition - By Larson, Hostetler and Edwards
- Calculus Concepts and Applications – by Paul A. Foerster

Online Resources:

- Albert io – online AP exam practice resource
- AP Central - Previously published and released AP questions
- Desmos – online graphing tool

Calculators:

- TI – 84 Plus
- TI - 89 Titanium

The following 21st century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments and projects:

21st Century skills:	**Mathematical Practices:**
• Critical thinking • Creativity • Collaboration • Communication • Information literacy • Technology literacy • Media literacy • Flexibility • Leadership • Initiative • Productivity • Social skills	• Make sense of problems and persevere in solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics • Use appropriate tools strategically • Attend to precision • Look for and make use of structure • Look for and express regularity in repeated reasoning

Students will write:

Explain the difference, as you understand it so far, between definite integral and indefinite integral.
The difference between a differential and a derivative.
The difference between mean value theorem and the intermediate value theorem.
What are the different kinds of Riemann sums?
What evidence can you think of to show that Riemann sums really do get to the value of a definite integral found by the fundamental theorem as *n* approaches infinity.
What are the different types of Indeterminate forms?
Explain how slope fields and numerical methods can be used to solve differential equations without finding an algebraic solution.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills.

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

Section: 4.1 Antiderivatives and Indefinite Integration

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Approximate Antiderivatives from Derivatives.
Teaching Objectives	• Recognize antiderivatives of basic functions.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 249 Foerster Text: p. 194

Section: 4.2 Area

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Discuss Archimedes method of exhaustion for finding area.
Teaching Objectives	• Interpret the definite integral as the limit of a Riemann sum.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 261 Foerster Text: p. 202

Section: 4.3 Riemann Sums and Definite Integral

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Model Riemann sum by drawing smaller rectangles and discuss trapezoid rule to approximate area under curve.
Teaching Objectives	• Express the limit of a Riemann sum in integral notation. • Approximate a definite integral
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p.272 Foerster Text: p. 209

Section: 4.4 The fundamental Theorem of Calculus

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Discuss notation difference between the definite and Indefinite Integral.
Teaching Objectives	• Calculate a definite integral using areas and properties of definite integrals. • Analyze functions defined by an integral. • Calculate antiderivatives
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 284 Foerster Text: p. 225

Section: 4.5 Integration by Substitution

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Show how the need for a new technique arises with the introduction of composite functions.
Teaching Objectives	• Evaluate definite integrals.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 297 Foerster Text: p. 232

Section: 4.6 Numerical Integration

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Revisit Trapezoid rule by comparing it to results from Riemann's Rectangular approximations where the bases of the rectangles equal the height of the trapezoids.
Teaching Objectives	• Evaluate definite integrals. • Apply definite integrals to problems involving motion.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 307 Foerster Text: p.232

Section: 5.1 The Natural Logarithmic Function: Differentiation

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Discuss the number *e* and its relationship to the natural logarithm.
Teaching Objectives	• Evaluate definite integrals.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 321 Foerster Text: p. 277

Section: 5.2 The Natural Logarithm Function: Integration

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Give examples of Integrating Rational Functions.
Teaching Objectives	• Evaluate definite integrals.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 330 Foerster Text: p. 294

Section: 5.3 Inverse Functions

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Give basic examples of finding inverse functions. Discuss relationship between domain and range for a function and its inverse.
Teaching Objectives	• Evaluate definite integrals.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 338 Foerster Text: p. 292

Section: 5.4 Exponential Functions

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Graph $f^{-1}(x)=e^x$ and $f(x)=\ln x$
Teaching Objectives	• Evaluate definite integrals.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 347 Foerster Text: p.293

Section: 5.5 Bases Other than *e* and Applications

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Discuss the bases system in general.
Teaching Objectives	• Evaluate definite integrals. • Interpret the meaning of a definite integral within a problem
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 357 Foerster Text: p.292-293

Section: 5.6 Differential Equations: Growth & Decay

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Discuss situations real world applications in medicine and science which require the modeling of growth and decay.
Teaching Objectives	• Evaluate definite integrals. • Interpret the meaning of a definite integral within a problem • Analyze differential equations to obtain general and specific solutions. • Interpret, create, and solve differential equations from problems in context.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 366 Foerster Text: p. 322

Section: 5.8 Inverse Trigonometric Functions: Differentiation

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Discuss domain and range of Sine and Cosine.
Teaching Objectives	• Evaluate definite integrals.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 386 Foerster Text: p. 152

Section: 5. 9 Inverse Trigonometric Functions: Integration

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Model how to derive the inverse trigonometric function.
Teaching Objectives	• Evaluate definite integrals.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 393 Foerster Text: p. 468

Section: 6.1 Area of a Region Between Two Curves

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Explain subtraction of the area of regions and how it will pertain to f(x)-g(x) where f(x)>g(x).
Teaching Objectives	• Apply definite integrals to problems involving area and volume. • Use the definite integral to solve problems in various contexts.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 418 Foerster Text: p. 240

Section: 6.2 Volume: The Disk Method

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Sketch the general concept behind the Disk Method and its relationship to the area of circles.
Teaching Objectives	• Apply definite integrals to problems involving area and volume. • Use the definite integral to solve problems in various contexts.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 428 Foerster Text: p. 247

Section: 6.3 Volume: The Shell Method

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Sketch the general concept behind the Shell Method and its relationship to the circumference of circles.
Teaching Objectives	• Apply definite integrals to problems involving area and volume. • Use the definite integral to solve problems in various contexts.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 437 Foerster Text: p. 399

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.
IDENTIFY BLOOM'S LEVELS.

Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

Accommodations/Modifications:

As per individual student's IEP or 504 plan.

Summative Assessments:

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- **Diagnostic Pre-Test**
- **Chapter Tests**
- **Projects**
- **End-Of –Course Assessment**

Accommodations/Modifications:

As per individual students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- **Projects**
- **Practice AP Exam Questions**
- **Homework**
- **Classwork**

Accommodations/Modifications:

As per individual student's IEP or 504 plan.

Black Horse Pike Regional School District Curriculum Template

ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21ST CENTURY GLOBAL SKILLS

Course Name: AP Calculus AB

Course Number: 0343000

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Course/Unit Title: AP Calculus AB **Grade Level(s):** **11-12**	**Unit Summary**: Students will prepare for the AP Calculus AB Exam by working both independently and in small groups on published exams. We will analyze each problem from the concepts being addressed, any distractions within the wording of the problem or answer choices, as well as common errors and misconceptions that could lead to a wrong answer. After the AP Exam, students will learn Integration by Parts and work on projects that highlight the real world applications of previously learned concepts in Calculus.
Essential Question(s): **How do I recognize key phrases and terms and then associate them with certain concepts in Calculus?** **Are there any basic errors that reoccur in my work as I practice solving published AP Exams?** **What is the algebraic rigor of the AP Calculus exam?** **What is the conceptual rigor of the AP Calculus exam?** **When would I need to use Integration by Parts?** **How would I use concepts like Derivatives and Integration to solve real world problems?**	Enduring Understanding(s): Students will be able to: • Recognize key phrases and associate them to specific concepts in Calculus • Verbalize a variety of strategies they can employ to monitor their own work for common errors that they have made in the past. • Understand the requirements of the AP Calculus College Board Exam in terms of conceptual rigor as opposed to algebraic rigor. • Solve a variety of problems using Integration by Parts. • Complete a selection projects that utilize some concept from Calculus.

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

Applications of the Mathematical Practices for AP Calculus:

Learning Target

MPAC 1 – Reasoning with definitions and theorems:
As students practice solving previously published AP Calculus exams, they will need to utilize their understanding of the definition of derivative in all its forms as well as all the applicable theorems we have studied.

MPAC 2 – Connecting concepts:
As students work through the AP Calculus test preparation packet and projects, they will need to make connections as to how continuity and differentiability are related as well as the various inverse relationships between concepts like differentiation, integration, as well as how mathematical concepts like derivative is modeled in many aspects in the real world through rates of change.

MPAC 3 – Implementing algebraic/computational processes:
Students will be tasked with reviewing their work to not only solve concepts algebraically but also review incorrect solutions to analyze the types of errors made.

MPAC 4 – Connecting multiple representations:
Studets will be asked to solve similar problems to see how they may be represented in a variety of formats both algebraic and graphic. They will also see problems written using a variety of similar notation.

MPAC 5 – Building notational fluency:
Students will be expected to write all solutions in correct notation from limits to derivatives to integration, and be know the difference and recognize each in their various forms.

MPAC 6 – Communicating:
Students will communicate their understanding through while class discussion, written response, and through their projects (which has them state information either in solution form or as a summary of the whole exploration, or both).

Inter-Disciplinary Connections:

Real World and Inter-disciplinary problems:
College Board & AP Central: Free response problems.

Students will engage with the following text, resources and tools:

Texts:

- Calculus of a Single Variable, Seventh Edition - By Larson, Hostetler and Edwards
- Calculus Concepts and Applications – by Paul A. Foerster

Online Resources:

- Albert io – online AP exam practice resource
- AP Central - Previously published and released AP questions
- Desmos – online graphing tool

Calculators:

- TI – 84 Plus
- TI - 89 Titanium

The following 21st century skills and the 8 mathematical practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments and projects:

21st Century skills:

- Critical thinking
- Creativity
- Collaboration
- Communication
- Information literacy
- Technology literacy
- Media literacy
- Flexibility
- Leadership
- Initiative
- Productivity
- Social skills

Mathematical Practices:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

Students will write:

Explain the difference, as you understand it so far, between definite integral and indefinite integral.
The difference between a differential and a derivative.
The difference between mean value theorem and the intermediate value theorem.
What are the different kinds of Riemann sums?
What evidence can you think of to show that Riemann sums really do get to the value of a definite integral found by the fundamental theorem as *n* approaches infinity.
What are the different types of Indeterminate forms?
Explain how slope fields and numerical methods can be used to solve differential equations without finding an algebraic solution.

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills.

Students will uncover and build skills through various classroom learning activities. Investigating algebra activities, modeling examples, using real-life application, using note-taking strategies, and using Smart Board technologies will all be explored as a blend of learning strategies to promote critical thinking, problem solving and performance skills of all learners. Other learning experiences could include alternative lesson openers, math and history applications, problem-solving workshops, interdisciplinary applications and projects.

Suggested warm-up activities, instructional strategies/activities, and assignments:

Section: Review for AP Exam

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Review Previous days assignments, teacher models solutions while students assess any errors and ask questions as needed.
Teaching Objectives	• Recognize key phrases and associate them to specific concepts in Calculus • Verbalize a variety of strategies they can employ to monitor their own work for common errors that they have made in the past. • Understand the requirements of the AP Calculus College Board Exam in terms of conceptual rigor as opposed to algebraic rigor.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Published Exams from College Board and AP Central

Section: 7.2 Integration by Parts

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Have students attempt a deceptively easy looking integration problem that will lead into the need for a new technique in order to solve.
Teaching Objectives	• Solve a variety of problems using Integration by Parts.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Larson Text: p. 494 Foerster Text: p. 480

Section: Project(s) [As time allows]

	Advanced Placement AB
Focus and Motivate **Starting Options (Lesson Warm Up)**	*Suggestions include but are not limited to:* Present the project and discuss applications as needed
Teaching Objectives	• Complete a selection projects that utilize some concept from Calculus.
Checking for Understanding	*Suggestions include but are not limited to:* Exit tickets (teacher made supplements) Inquiry Formative Assessment
Practice and Apply **Assigning Homework**	Teacher Provided Project as needed.

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.
IDENTIFY BLOOM'S LEVELS.

Formative Assessments:

The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.

Accommodations/Modifications:

As per individual student's IEP or 504 plan.

Summative Assessments:

The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

- **Diagnostic Pre-Test**
- **Chapter Tests**
- **Projects**
- **End-Of –Course Assessment**

Accommodations/Modifications:

As per individual students' IEP or 504 plan.

Performance Assessments:

The following assessments require students to utilize various strands of mathematics.

- **Projects**
- **Practice AP Exam Questions**
- **Homework**
- **Classwork**

Accommodations/Modifications:

As per individual student's IEP or 504 plan.

Mathematical Practices for AP Calculus (MPACs)

The Mathematical Practices for AP Calculus (MPACs) capture important aspects of the work that mathematicians engage in, at the level of competence expected of AP Calculus students. They are drawn from the rich work in the National Council of Teachers of Mathematics (NCTM) Process Standards and the Association of American Colleges and Universities (AAC&U) Quantitative Literacy VALUE Rubric.

Embedding these practices in the study of calculus enables students to establish mathematical lines of reasoning and use them to apply mathematical concepts and tools to solve problems. The MPACs are not intended to be viewed as discrete items that can be checked off a list; rather, they are highly interrelated tools that should be utilized frequently and in diverse contexts.

The sample exam questions included in the AP Calculus AB and AP Calculus BC Course and Exam Description demonstrate various ways the learning objectives can be linked with the MPACs.

MPAC 1: Reasoning with definitions and theorems

Students can:

- Use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results.
- Confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem.
- Apply definitions and theorems in the process of solving a problem.
- Interpret quantifiers in definitions and theorems (e.g., "for all," "there exists").
- Develop conjectures based on exploration with technology.
- Produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

MPAC 2: Connecting concepts

Students can:

- Relate the concept of a limit to all aspects of calculus.
- Use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems.
- Connect concepts to their visual representations with and without technology.
- Identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

- Select appropriate mathematical strategies.
- Sequence algebraic/computational procedures logically.
- Complete algebraic/computational processes correctly.
- Apply technology strategically to solve problems.
- Attend to precision graphically, numerically, analytically, and verbally and specify units of measure.
- Connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

- Associate tables, graphs, and symbolic representations of functions.
- Develop concepts using graphical, symbolical, verbal, or numerical representations with and without technology.
- Identify how mathematical characteristics of functions are related in different representations.
- Extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values).
- Construct one representational form from another (e.g., a table from a graph or a graph from given information).
- Consider multiple representations (graphical, numerical, analytical, and verbal) of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building notational fluency

Students can:

- Know and use a variety of notations.
- Connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum).
- Connect notation to different representations (graphical, numerical, analytical, and verbal).
- Assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating

Students can:

- Clearly present methods, reasoning, justifications, and conclusions.
- Use accurate and precise language and notation.
- Explain the meaning of expressions, notation, and results in terms of a context (including units).
- Explain the connections among concepts.
- Critically interpret and accurately report information provided by technology.
- Analyze, evaluate, and compare the reasoning of others.

Source: https://apcentral.collegeboard.org/courses/resources/mathematical-practices-ap-calculus-mpacs